



Western Australian Certificate of Education Sample Examination, 2016

Question/Answer Booklet

MATHEMATICS METHODS

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures

| | | | | | | | | |
|--|--|--|--|--|--|--|--|--|
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In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

Number of additional answer booklets used (if applicable):

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|------------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|--------------------|
| Section One: Calculator-free | 9 | 9 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 92 | 65 |
| Total | | | | | 100 |

Instructions to candidates

- The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed

65% (92 Marks)

This section has **12** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Suggested working time: 100 minutes.

Question 10

(6 marks)

Certain medical tests require the patient to be injected with a solution containing 0.5 micrograms (μg) of the radioactive material Technetium-99. This material decays according to the rule:

$$T = T_0 e^{-0.1155t} \text{ where } t \text{ is the time (in hours) from injection.}$$

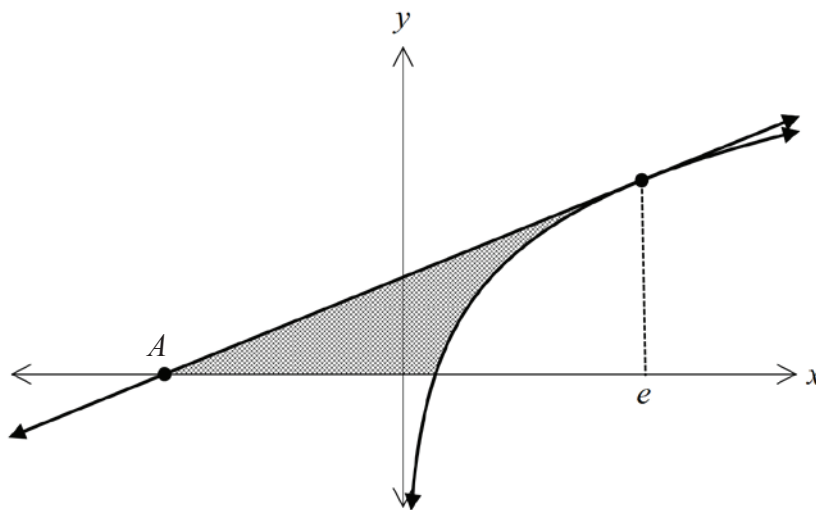
- (a) What is the value of T_0 ? (1 mark)
- (b) What is the half-life of Technetium-99? (2 marks)
- (c) After how long is the amount of Technetium-99 left in the patient's system less than 1% of the initial amount? Give your answer to the nearest hour. (3 marks)

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Question 11

(9 marks)

The diagram below shows the graph of the function $f(x) = \ln x + 1$ and a linear function $g(x)$, which is a tangent to $f(x)$. When $x = e$, $g(x) = f(x)$.



- (a) Determine $g(x)$, the equation of the tangent. (3 marks)

- (b) Determine the exact coordinates of A , the point where $g(x)$ intersects the x -axis. (1 mark)

- (c) Verify that $f(x)$ cuts the x -axis at the point $\left(\frac{1}{e}, 0\right)$. (1 mark)

- (d) Determine the area of the shaded region enclosed by $f(x)$, $g(x)$ and the x -axis.
(4 marks)

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Question 12

(12 marks)

Rebecca sells potatoes at her organic fruit and vegetable shop that have weights normally distributed with a mean of 230 g and a standard deviation of 5 g.

- (a) Determine the probability that one of Rebecca's potatoes, selected at random, will weigh between 223 g and 235 g. (1 mark)
- (b) Five percent of Rebeca's potatoes weigh less than w g. Determine w to the nearest gram. (2 marks)
- (c) A customer buys twelve potatoes.
- (i) Determine the probability that all twelve potatoes weigh between 223 g and 235 g. (2 marks)
- (ii) If the customer is selecting the twelve potatoes one at a time, determine the probability that it takes the selection of eight potatoes before six potatoes weighing between 223 g and 235 g have been selected. (3 marks)

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Rebecca also sells oranges. The weights of these oranges are also normally distributed. It is known that 5% of the oranges weigh less than 153 g while 12% of the oranges weigh more than 210 g.

- (d) Determine the mean and standard deviation of the weights of the oranges. (4 marks)

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Question 13

(5 marks)

The decibel scale for sound, measured in decibels (dB), is defined as: $D = 20 \log_{10} \left(\frac{P}{P_{ref}} \right)$,

where P is the pressure of the sound being measured and P_{ref} is a fixed reference pressure.

- (a) What is the decibel measure for a sound with pressure $2P_{ref}$? (1 mark)
- (b) The sound produced by a symphony orchestra measures 120 dB, while that of a rock concert measures 150 dB. How many times greater is the sound pressure of the rock concert than that of the orchestra? (4 marks)

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Question 14

(10 marks)

(a) The discrete random variable X has the following probability distribution:

| | | | | | |
|------------|-----|-----|-----|------|-----|
| x | 1 | 2 | 3 | 4 | 5 |
| $P(X = x)$ | 0.1 | a | 0.3 | 0.25 | b |

(i) Determine the values of a and b if the expected value, $E(X) = 3.3$. (3 marks)

(ii) Determine the variance, $Var(x)$. (2 marks)

(iii) State the value of $Var(X + 5)$. (1 mark)

(iv) State the value of $Var(2X + 5)$. (1 mark)

(b) Daniel has been offered a sales position at a car yard. His weekly pay will comprise two components, a retainer of \$250 and a commission of \$400 for each new car sold. The following table shows the probability of his selling specific numbers of cars each week.

| | | | | | |
|------------|-----|-----|------|------|------|
| n | 0 | 1 | 2 | 3 | 4 |
| $P(N = n)$ | 0.3 | 0.4 | 0.25 | 0.04 | 0.01 |

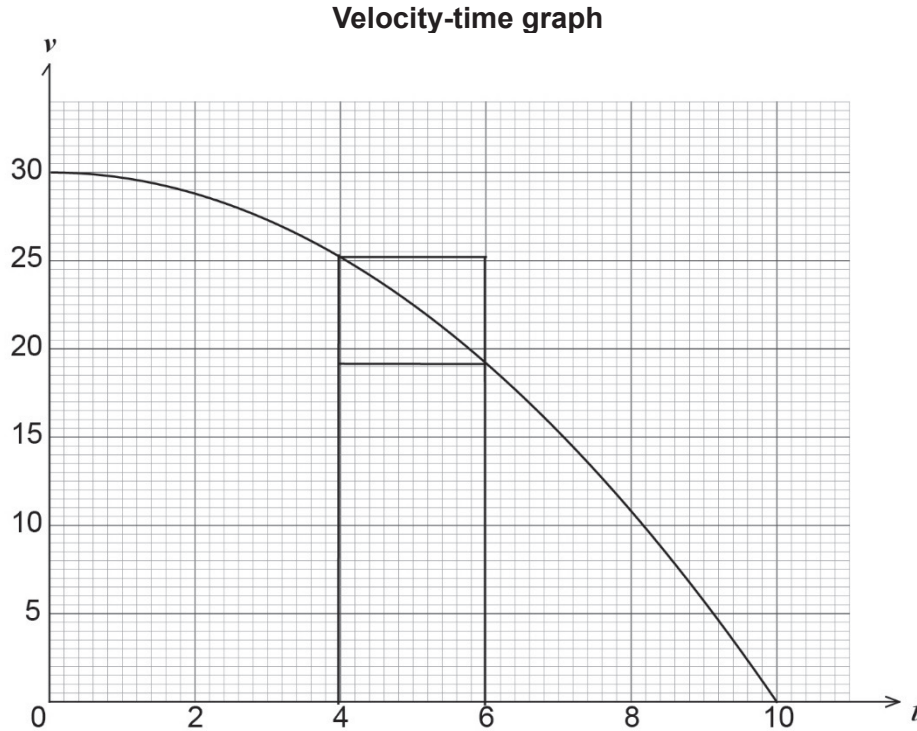
Calculate Daniel's expected weekly pay. (3 marks)

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Question 15

(6 marks)

A train is travelling at 30 metres per second when the brakes are applied. The velocity of the train is given by the equation $v = 30 - 0.3t^2$, where t represents the time in seconds after the brakes are applied.



The area under a velocity-time graph gives the total distance travelled for a particular time period.

- (a) Complete the tables below and estimate the distance travelled by the train during the first six seconds by calculating the mean of the areas of the circumscribed and inscribed rectangles. (The rectangles for the 4–6 seconds interval are shown on the grid above.)

| | | | | |
|----------------------------------|---|------|---|------|
| Time (t) | 0 | 2 | 4 | 6 |
| Velocity (v) | | 28.8 | | 19.2 |

| | | | | |
|---------------------------|-----|-----|------|-------|
| Rectangle | 0–2 | 2–4 | 4–6 | Total |
| Circumscribed area | | | 50.4 | |
| Inscribed area | | | 38.4 | |

Estimate of total distance travelled _____ metres. (5 marks)

- (b) Describe how you could better estimate the distance travelled by the train during the first six seconds than by the method used in part (a). (1 mark)

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Question 16**(4 marks)**

Roland spends X hours writing poetry during the day.

The probability distribution of X is given by:

$$f(x) = \begin{cases} 2(1-x) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Evaluate $E(X)$, the expected value of X , to the nearest minute. **(2 marks)**

(b) Determine the variance of X . **(2 marks)**

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Question 17

(8 marks)

A random sample of 100 people indicated that 19% had taken a plane flight in the last year.

- (a) Determine a 90% confidence interval for the proportion of the population that had taken a plane flight in the last year. (2 marks)

Assume the 19% sample proportion applies to the whole population.

- (b) A new sample of 200 people was taken and X = the number of people who had taken a plane flight in the last year was recorded. Give a range, using the 95% confidence interval, within which you would expect X to lie. (1 mark)
- (c) Determine the probability that in a random sample of 120 people, the number who had taken a plane flight in the last year was greater than 26. (3 marks)

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Question 18

(10 marks)

A random survey was conducted to estimate the proportion of mobile phone users who favoured smart phones over standard phones. It was found that 283 out of 412 people surveyed preferred a smart phone.

- (a) Determine the sample proportion \hat{p} of those in the survey who preferred a smart phone. (1 mark)
- (b) Use the survey results to estimate the standard deviation of \hat{p} . (2 marks)
- (c) A follow-up survey is to be conducted to confirm the results of the initial survey. Working with a confidence interval of 95%, estimate the sample size necessary to ensure margin of error of at most 4%. (3 marks)

The 90% confidence interval of the sample proportion \hat{p} from the initial survey is $0.649 \leq \hat{p} \leq 0.725$.

- (d) Use the 90% confidence interval of the initial sample to compare the following samples:
- (i) A random sample of 365 people at a shopping centre found that 258 had a preference for a smart phone. (2 marks)
- (ii) A random sample of 78 people at a retirement village found that 32 had a preference for a smart phone. (2 marks)

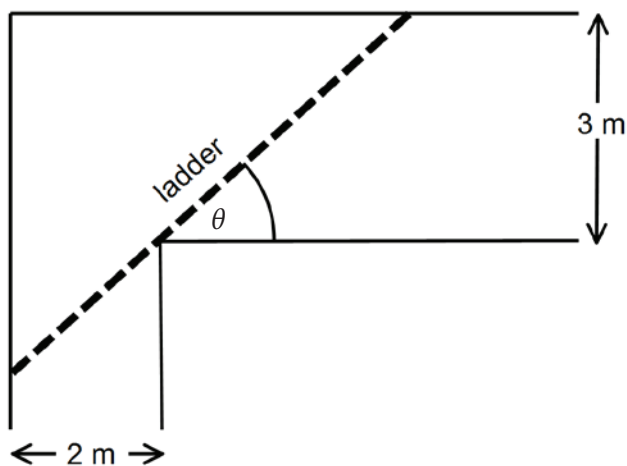
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Question 19

(7 marks)

Two corridors meet at right angles and are 3 m and 2 m wide respectively. The angle between the wall and the ladder is marked on the diagram as θ .

A ladder (of negligible width) is to be carried horizontally along this L-shaped space by two workers. The workers need to know the length of the longest ladder that can be carried around this corner.



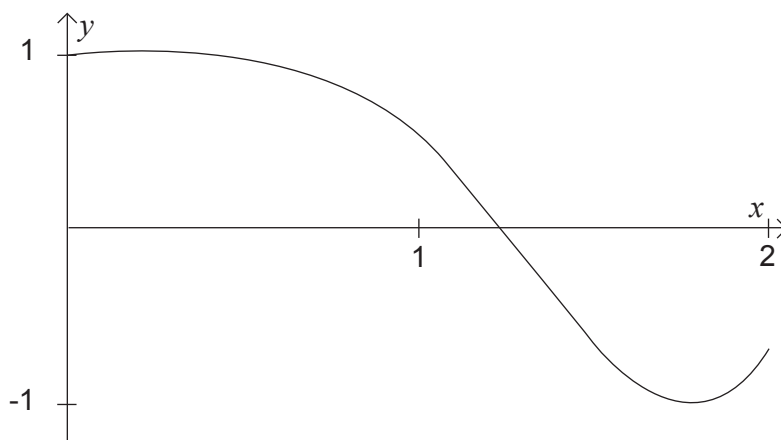
- (a) Show that the length of the ladder (L) is given by $L = \frac{2}{\cos\theta} + \frac{3}{\sin\theta}$. (3 marks)

- (b) The workers' ladder is 6.5 m long. Will they be able to carry their ladder along this L-shaped space? Justify your answer. (4 marks)

Question 20

(10 marks)

The graph of the function $f(x)=\cos x^2$ for $0 \leq x \leq 2$ is provided below.

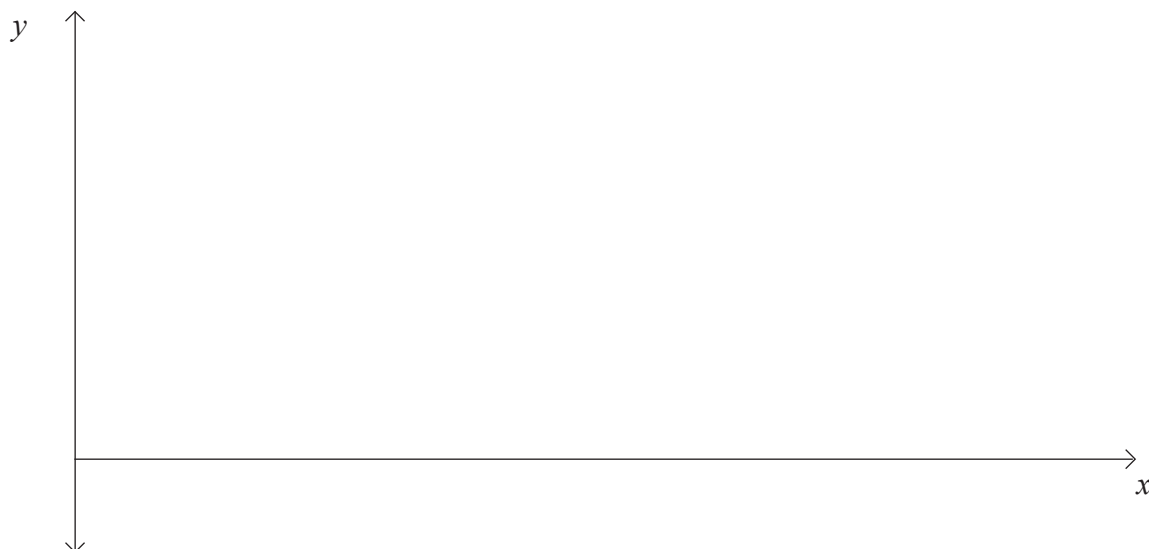


- (a) The function $A(x)$ is defined as $A(x) = \int_0^x f(t)dt$, for $0 \leq x \leq 2$. Determine the value of x when $A(x)$ starts to decrease. (3 marks)

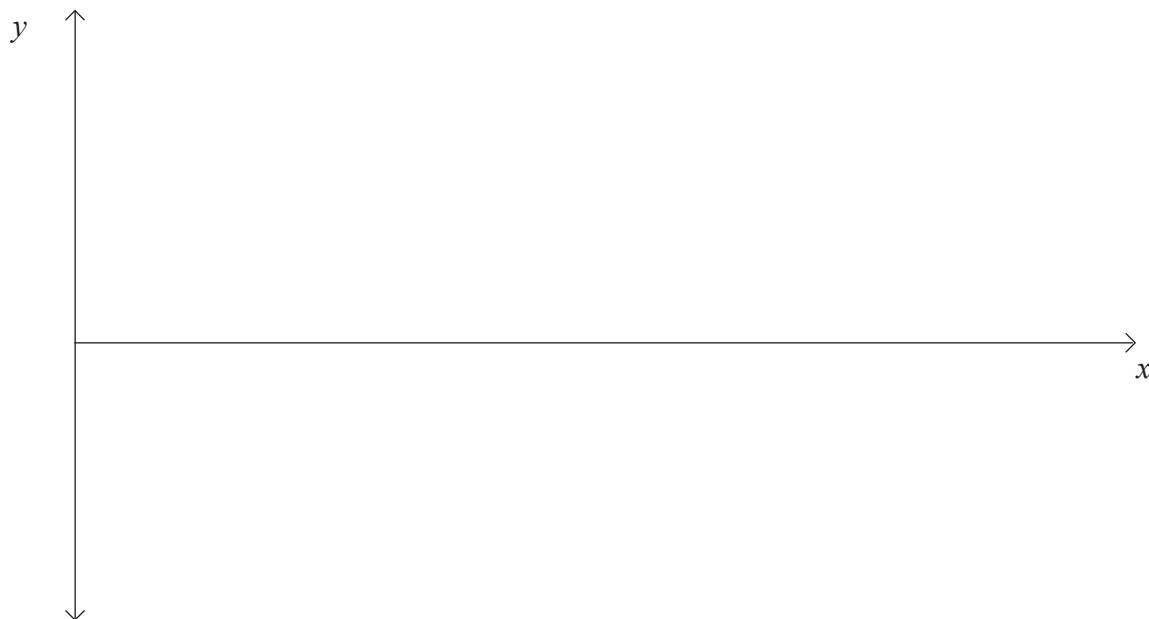
- (b) Complete the table below. (2 marks)

| | | | | | | | | | | |
|--------|-------|-------|-----|-------|-------|-------|-----|-----|-----|-------|
| x | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| $A(x)$ | 0.200 | 0.399 | | 0.768 | 0.905 | 0.974 | | | | 0.461 |

- (c) On the axes below, plot the values from the table in part (b), and hence sketch the graph of $A(x)$ for $0 \leq x \leq 2$. (2 marks)



- (d) Use your graph from part (c) to sketch the graph of $A'(x)$ on the axes below. (2 marks)



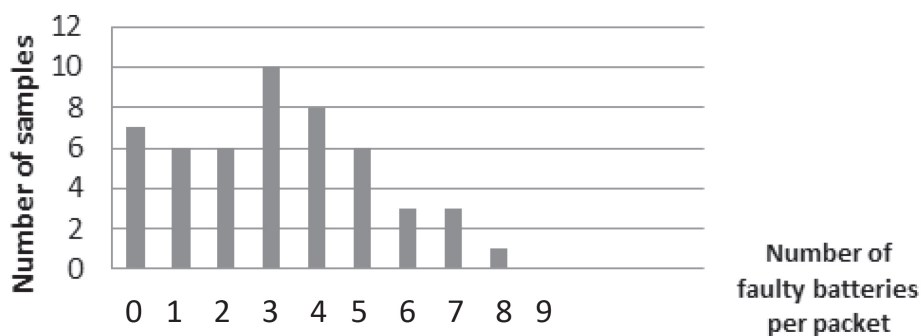
- (e) Based on your observations of the graphs in this question, make a conjecture about the defining rule for $A'(x)$. (1 mark)

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Question 21

(5 marks)

The graph below shows the number of faulty batteries per packet of 50 AAA batteries, when 50 packets are sampled at random.



- (a) Identify the type of distribution of X = the number of faulty batteries per packet of 50 AAA batteries. (1 mark)

A manufacturer of AAA batteries assumes that 99% of the batteries produced are fault-free. Ten samples of 50 packets of 50 AAA batteries are selected at random and tested. The number of faulty batteries in each of the 10 random samples is shown below.

| Sample | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------------------|----|----|----|----|----|----|----|----|----|----|
| Number of faulty batteries | 34 | 28 | 22 | 28 | 28 | 30 | 22 | 28 | 28 | 30 |

- (b) Using the assumption that 99% of batteries are fault free calculate the 95% confidence interval for the proportion of faulty batteries expected when sampling. (3 marks)
- (c) Decide which of the samples, if any lie outside the 95% confidence level. (1 mark)

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Additional working space

Question number: _____

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